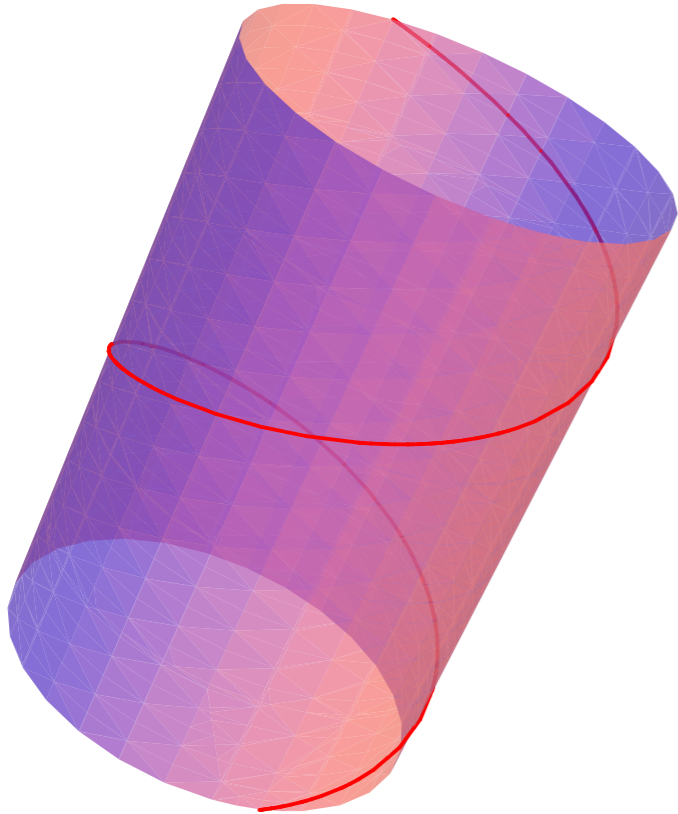
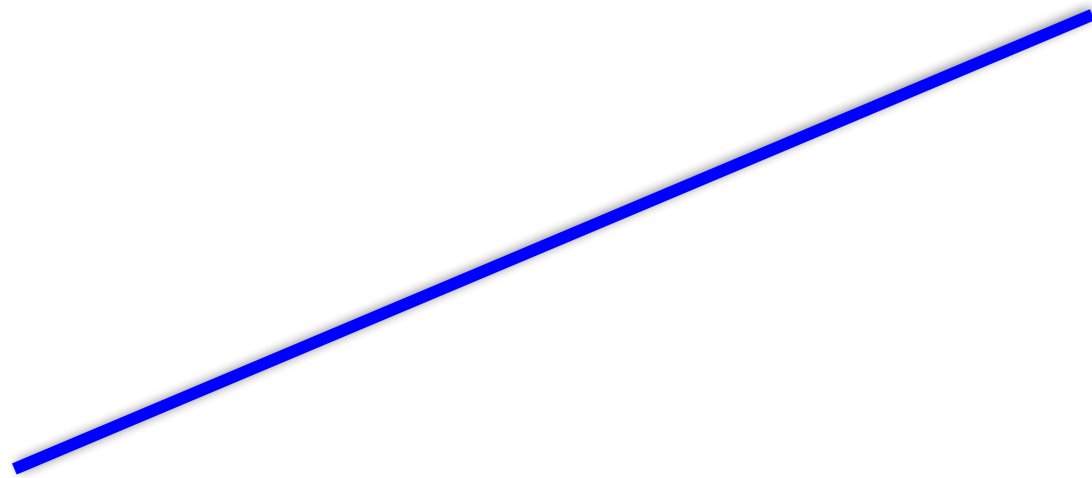


What is Curvature?

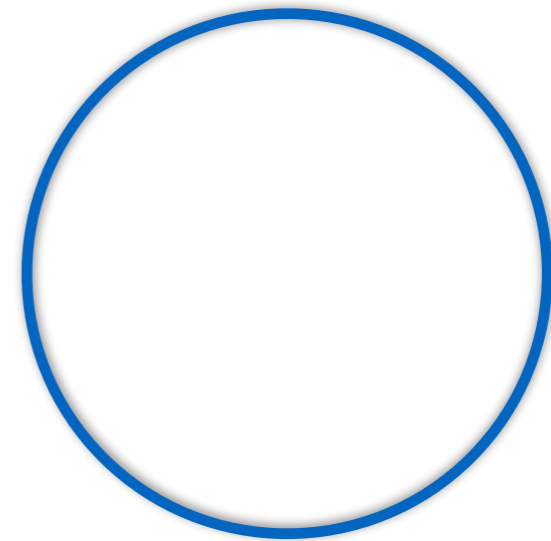


Curvature of Simple Curves

Rough Idea: Curvature measures not being flat.

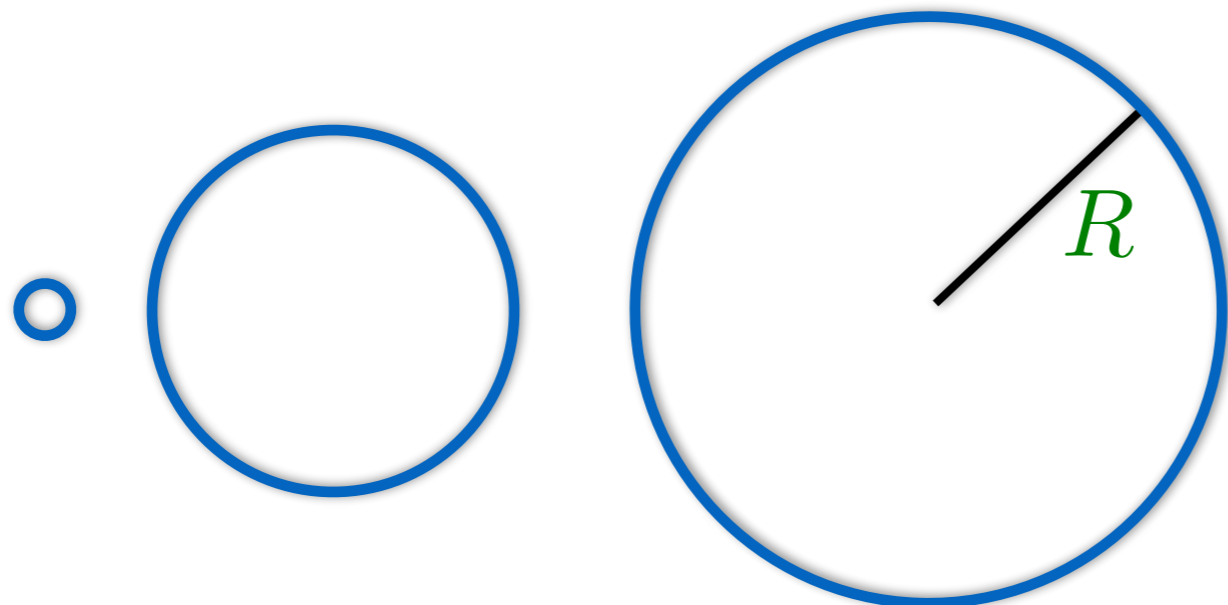


Curvature = 0



Curvature $\neq 0$

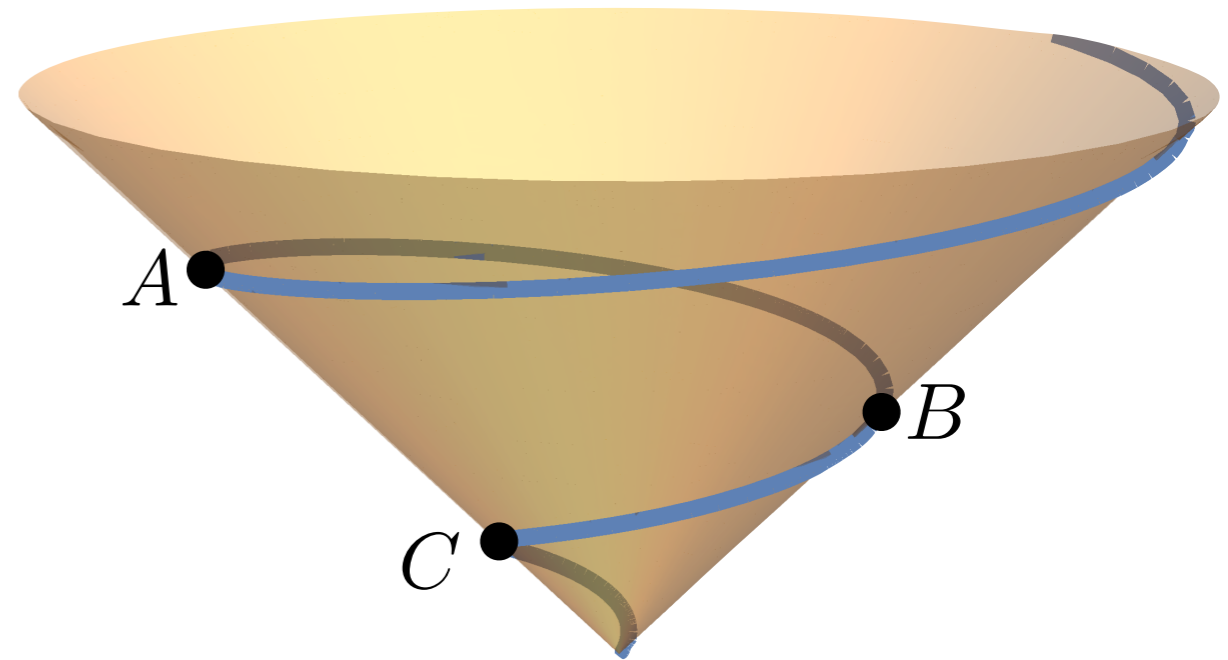
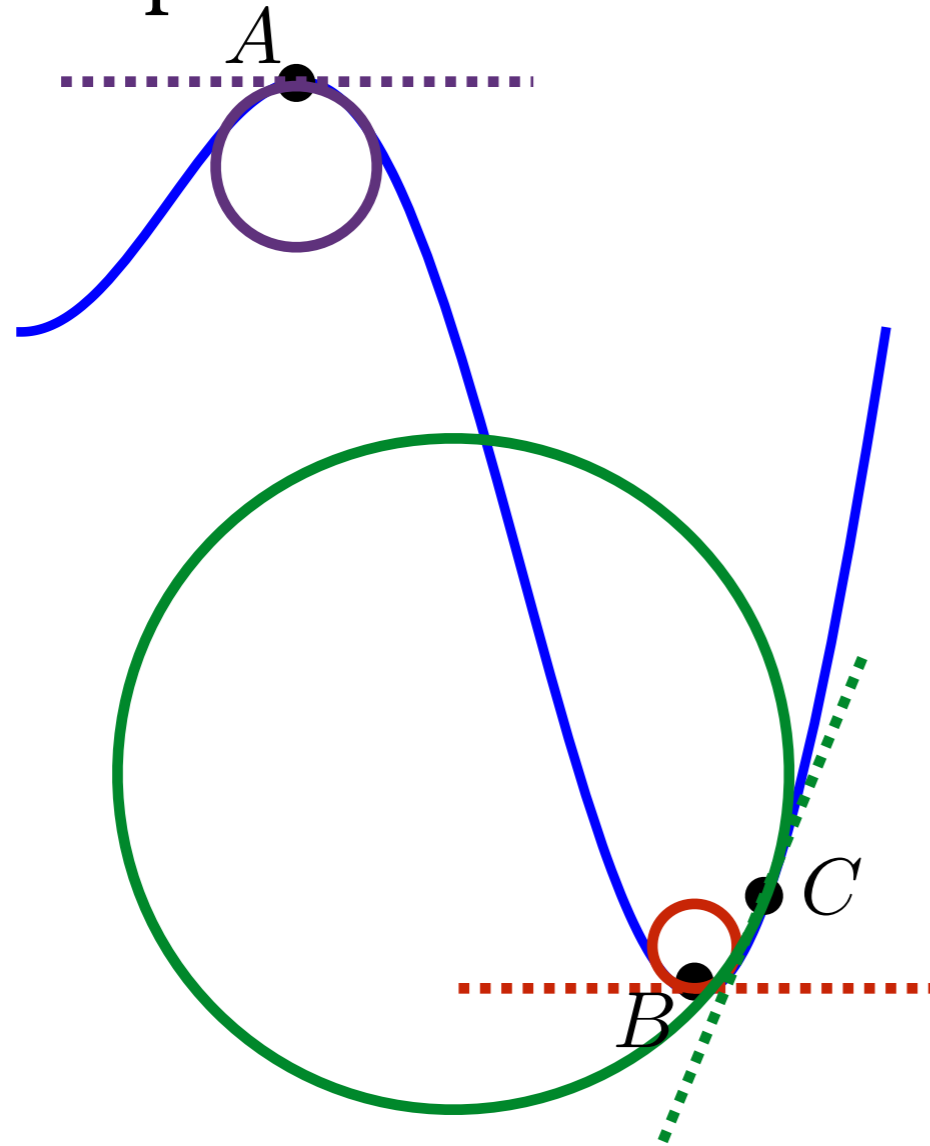
Which is more curved?



Curvature = $\frac{1}{R}$

Curvature of Exotic Curves

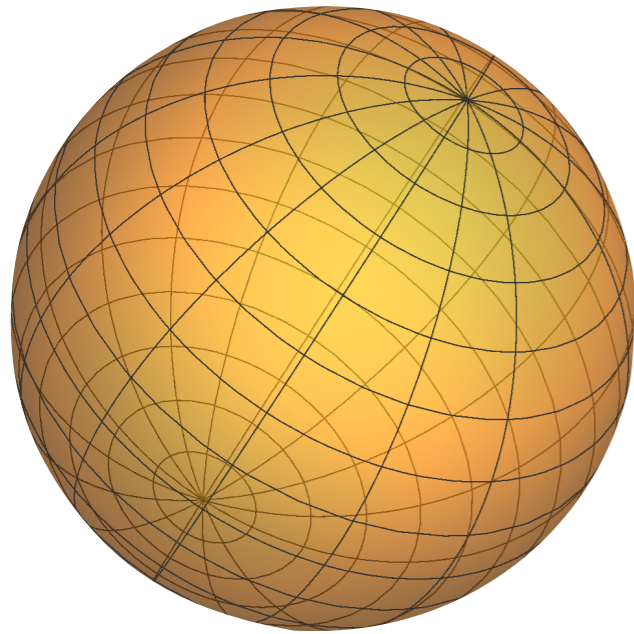
At what points is the curvature highest?



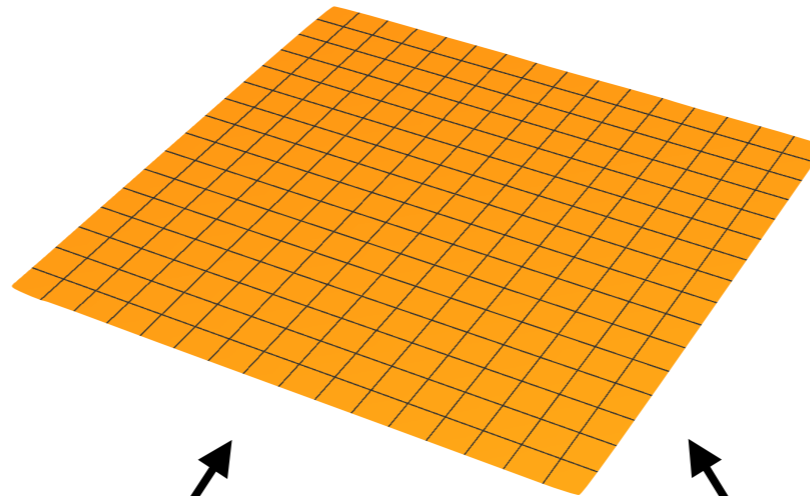
The curvature at a point on a curve is the curvature of the closest approximating circle at that point.

Curvature of Surfaces

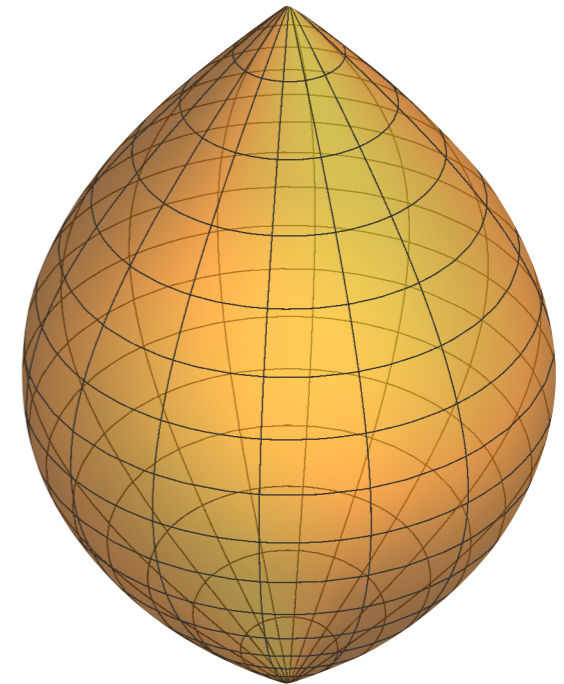
Rough Idea: Curvature measures not being flat. Which surfaces are flat?



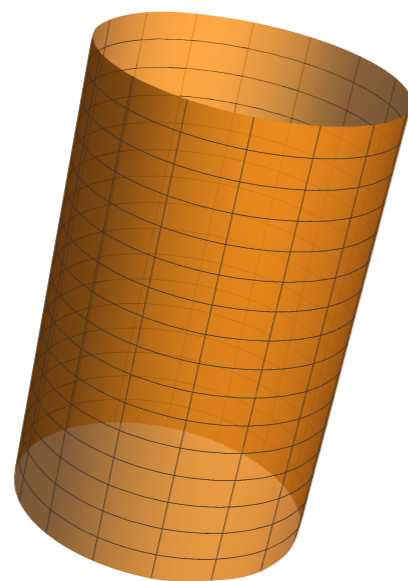
Not Flat



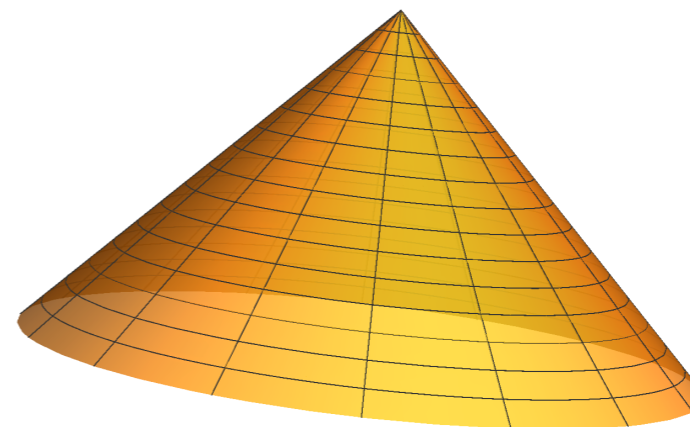
Flat



Not Flat



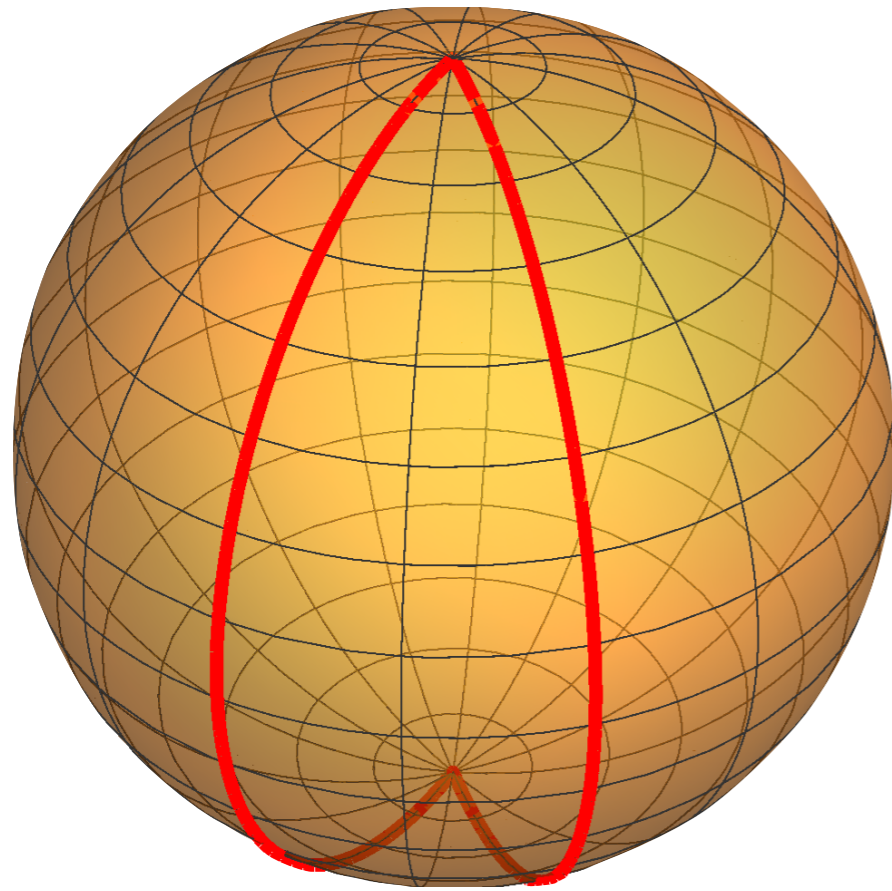
Flat



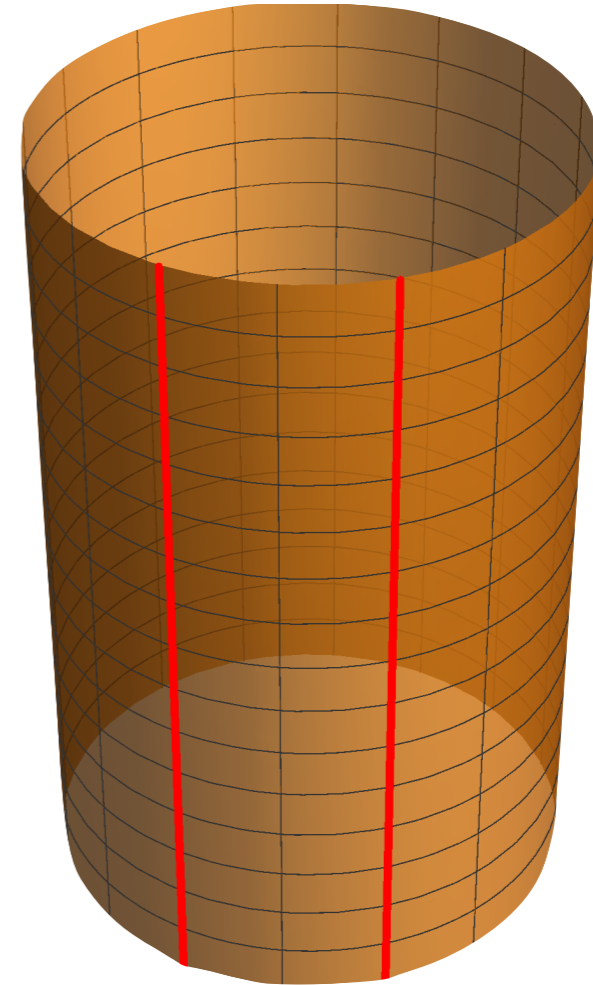
Flat

What Does it Mean to be Flat?

A surface is flat if straight lines on the surface remain parallel.

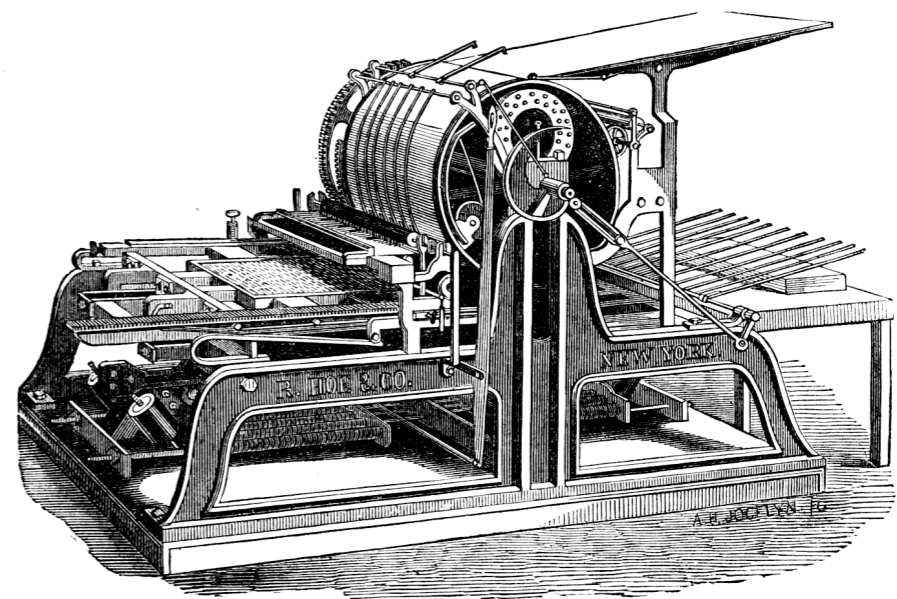
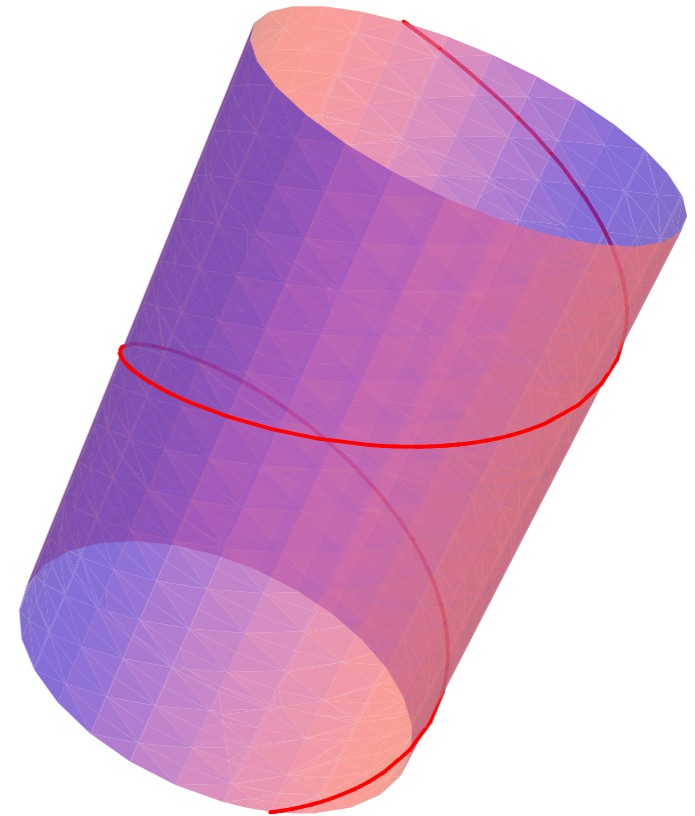
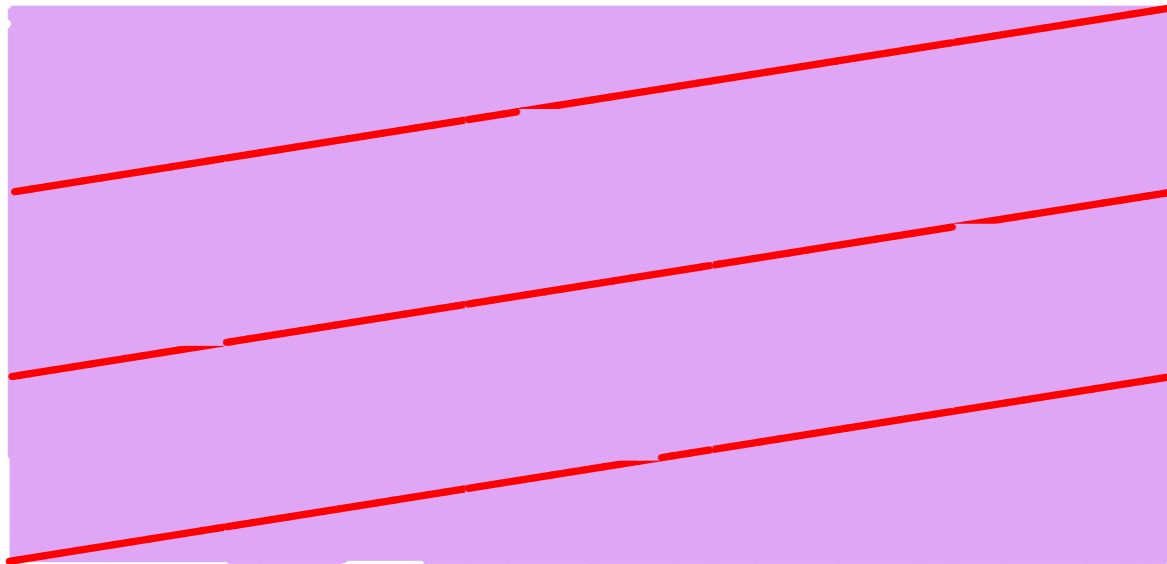


Not Flat



Flat

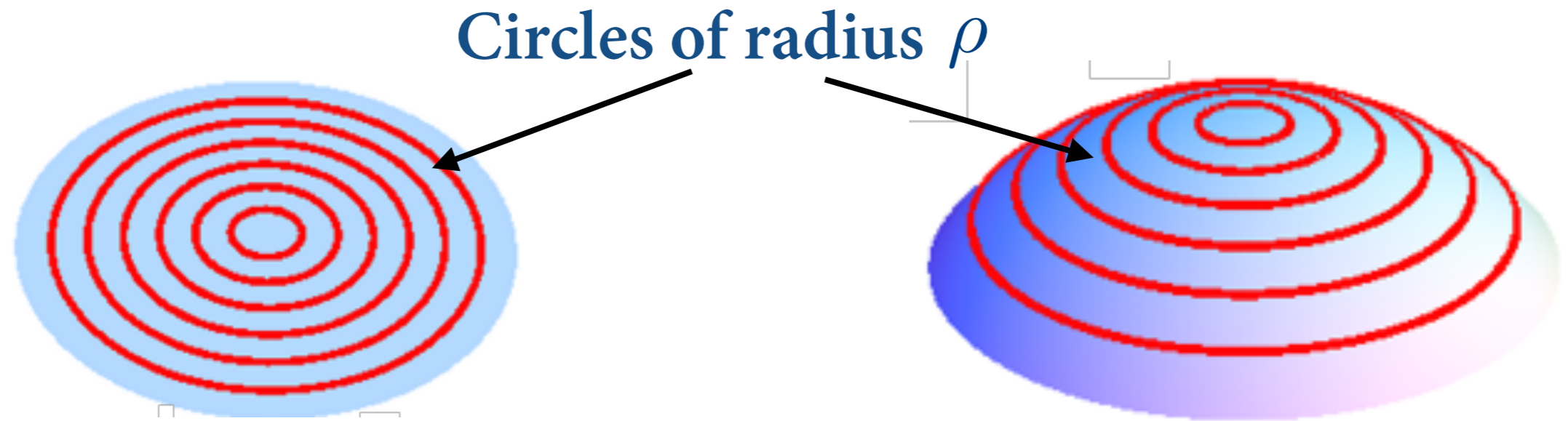
Flat Geometry



HOE'S ONE-CYLINDER PRINTING PRESS.

Printing Press

Gaussian Curvature



$$\text{Circumference} = 2\pi\rho$$

$$\text{Circumference} = C(\rho) < 2\pi\rho$$

The Gaussian curvature K measures the change in circumference on the surface:

$$\lim_{\rho \rightarrow 0} 3 \frac{2\pi\rho - C(\rho)}{\pi\rho^3}$$

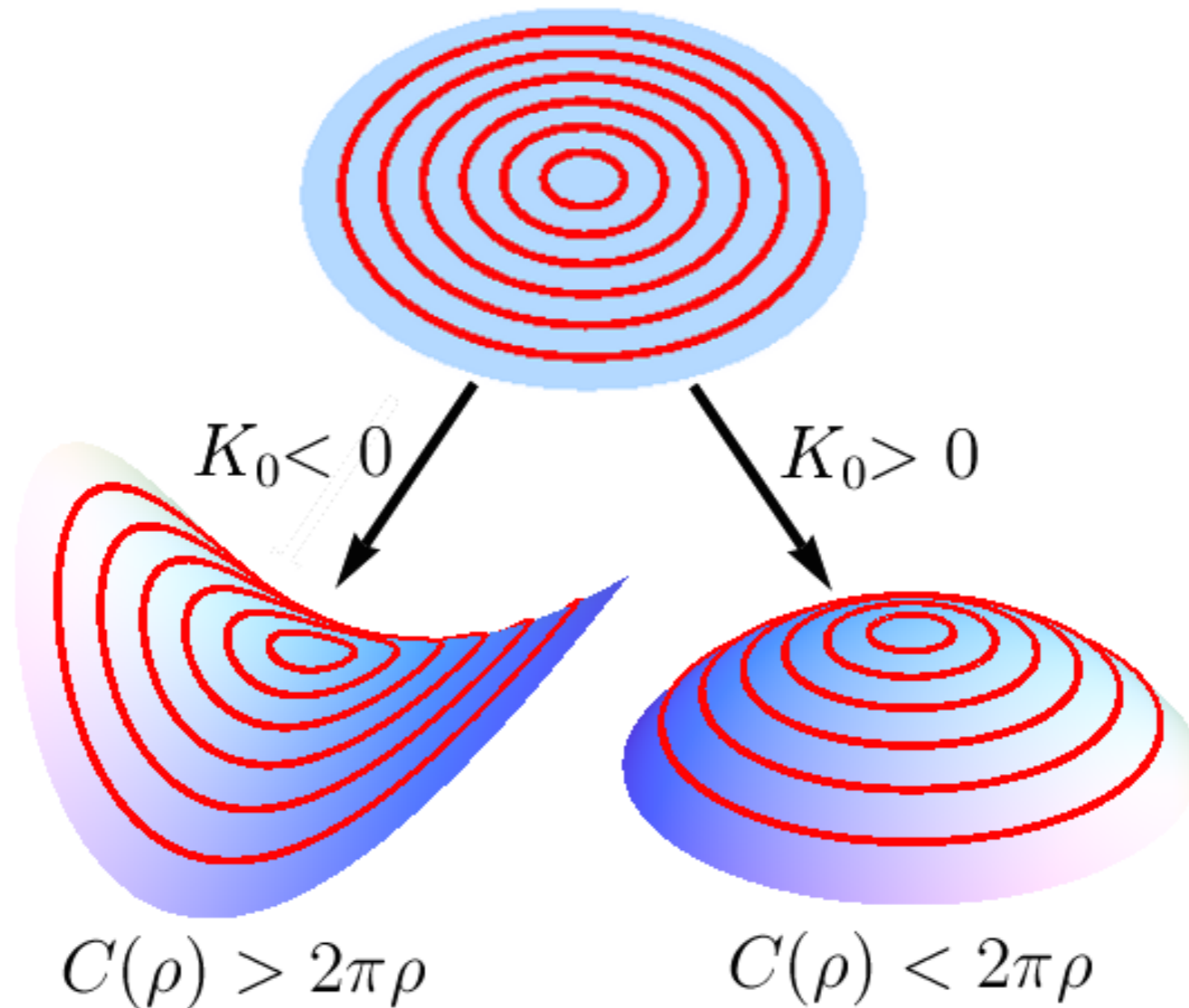
Flat surfaces have zero Gaussian curvature!

A Sphere is not Flat



There is no perfect map of the Earth!

Curvature Can be Negative



$$K = \lim_{r \rightarrow 0} 3 \frac{2\pi\rho - C(\rho)}{\pi\rho^3}$$

Wavy Surfaces have Negative Curvature

